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## C.U.SHAH UNIVERSITY

WADHWAN CITY

University (Winter) Examination -2013
Course Name: M.Tech(EC) Sem-I Subject : Information Theory \& Coding
Marks :70
Duration :- 2:30 Hours
Date : 06/01/2014

## Instruction

(1) Attempt all Questions of both sections in same answer book / Supplementary.
(2) Use of Programmable calculator \& any other electronic instrument is prohibited.
(3) Instructions written on main answer Book are strictly to be obeyed.
(4)Draw neat diagrams \& figures (If necessary) at right places.
(5) Assume suitable \& Perfect data if needed

## SECTION - I

Q. 1 (a) Explain Conditional Probability.02
(b) Explain Probability Density Function. ..... 02
(c) Explain uniquely decodable code with suitable example. ..... 02
(d) What is Independent events? ..... 01
Q. 2 (a) State and prove the Bay's theorem. ..... 05
(b) If X and Y are uncorrelated and have zero means than prove that ..... 05$\mathrm{E}\left\{(\mathrm{X}+\mathrm{Y})^{2}\right\}=\mathrm{E}\left(\mathrm{X}^{2}\right)+\mathrm{E}\left(\mathrm{Y}^{2}\right)$.
(c) What is an Instantaneous code? Describe the procedure to construct a 04 binary instantaneous code.
Q. 2 (a) State and prove the Kraft's Inequality.05
(b) Find the CDF of the Gaussian Random Variable whose PDF is given by 05

$$
f_{X}(x)=\frac{1}{\sqrt{2 \pi \sigma}} e^{-(x-\mu)^{2}} \mathrm{~L}^{2 \sigma^{2}}
$$

(c) The PDF of amplitude X of a certain signal $\mathrm{x}(\mathrm{t})$ is given by 04 $P_{x}(x)=0.5|x| \cdot e^{-|x|}$.
Find the probability that (i) $x \geq 1$. (ii) $-1 \leq x \leq 2$.
Q. 3 (a) Derive the mean and variance of Exponential random variable whose PDF 07 is given by $f_{X}(x)=\lambda e^{-\lambda x}$.
(b) The Joint PDF of random variable X and Y is given by 07 $P_{x y}(x, y)=K \cdot e^{-\left(x^{2}+x y+y^{2}\right)}$
Determine (i) the constant K (ii) $P_{x}(x)$ (iii) $P_{y}(y)$.

## OR

Q. 3 (a) Derive the mean and variance of Poisson random variable whose PDF is 07 given by $f_{X}(x)=e^{-\lambda} \frac{\lambda^{K}}{K!}$.
(b) Write short note on Ergodic Process.

## SECTION - II

Q. 4 (a) Explain the source Entropy. 02
(b) Define (i) Information Rate (ii) Code Rate. 02
(c) Explain the Binary Symmetric Channel. 02
(d) What is conditional entropy? 01
Q. 5 (a) Encode the sequence "BADF" using Arithmetic coding for the symbols 05 with following probability distribution:

| Symbol | A | B | C | D | E | F | G | H |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Length | 0.1 | 0.2 | 0.1 | 0.3 | 0.05 | 0.1 | 0.05 | 0.1 |

(b) Write short note on Mutual Information. 05
(c) An analog signal is bandlimited to 10 KHz is quantized in 8 levels of PCM 04 system with probabilities of $1 / 4,1 / 5,1 / 5,1 / 10,1 / 10,1 / 20,1 / 20$ and $1 / 20$ respectively. Find the entropy and rate of information. OR
Q. 5 (a) Write short note on Cyclic codes.
(b) For a $(6,3)$ systematic linear block code, the three parity check digits are $\mathrm{c} 4=\mathrm{d} 1+\mathrm{d} 2+\mathrm{d} 3, \quad \mathrm{c} 5=\mathrm{d} 1+\mathrm{d} 2, \quad \mathrm{c} 5=\mathrm{d} 14+\mathrm{d} 8$ s
i) Construct the appropriate generator matrix for this code and code table.
ii) Determine the error correcting capability.
iii)Decode the received words $101100,000110,101010$.
(c) Verify the following expression $\mathrm{H}(\mathrm{X} ; \mathrm{Y})=\mathrm{H}(\mathrm{X} / \mathrm{Y})+\mathrm{H}(\mathrm{Y})$
Q. 6 (a) A source emits seven messages with probabilities $1 / 2,1 / 4,1 / 8,1 / 16,1 / 32$, $1 / 64$, and $1 / 64$, respectively. Find the entropy of the source. Obtain the compact binary code and find the average length of the code word. Determine the efficiency and the redundancy of the code.
(b) A binary channel matrix is given by

$$
\begin{aligned}
& x_{1} \\
& x_{2}
\end{aligned}\left[\begin{array}{cc}
2 / 31 & 1 / 3 \\
1 / 10 & 9 / 10
\end{array}\right] \quad \begin{gathered}
x 1, x 2=\text { input } \\
y 1, y 2=\text { output }
\end{gathered}
$$

$P x(x 1)=1 / 3$ and $P(x 2)=2 / 3$. Determine $H(X), H(X / Y), H(Y), H(Y / X)$ and I(X; Y).

## OR

Q. 6 (a) A source emits seven messages with probabilities $1 / 3,1 / 3,1 / 9,1 / 9,1 / 27$, $1 / 27$, and $1 / 27$, respectively. Find the entropy of the source. Obtain the compact 3 -ary code and find the average length of the code word. Determine the efficiency and the redundancy of the code.
(b) Explain Viterbi Decoding Algorithm in detail.

